



# Effect of non-uniform heat flux on wall friction and convection heat transfer coefficient in a trapezoidal channel

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Received 15 March 2000; received in revised form 1 September 2000

## Abstract

Turbulent wall friction and forced convection heat transfer in a water-cooled trapezoidal channel were experimentally investigated. The parameter ranges were: Reynolds number =  $10^4$ – $8.6 \times 10^4$ , and Prandtl number = 2.2–5.5. The channel hydraulic diameter was 1.14 cm, and its total and heated lengths were 139.4 and 60.9 cm, respectively. The Colebrook correlation predicted the unheated channel friction factors well. It, however, systematically over predicted the measured friction factors obtained with the test section non-uniformly heated. Perimeter-average convective heat transfer coefficients for laterally non-uniform imposed heat fluxes were compared with the predictions of three widely-used correlations for turbulent flow in circular channels. All the correlations under predicted the data, typically by 11–28%. © 2001 Elsevier Science Ltd. All rights reserved.

*Keywords:* Turbulence; Friction; Trapezoidal; Convection

## 1. Introduction

Turbulent friction factor and heat transfer in non-circular channels have been studied rather extensively. The published studies include [1,2] for uniformly heated rectangular channels; [3–6] for triangular channels; [7–10] for ellipsoidal channels; and [11,12] for trapezoidal channels. Conflicting results on the suitability of using commonly applied circular channel correlations with an appropriately defined hydraulic diameter have been reported, furthermore. The use of circular channel correlations, along with hydraulic diameter approximation, appears to be adequate for less extreme non-circular cross-sections. Sharp corners decrease the average heat transfer coefficient by decreasing the local heat transfer coefficients in those areas; and increasing the Reynolds number and rounding the sharp corners appear to decrease this effect. Little work has been reported on the effect of non-uniform heating on the

variation of the local heat transfer coefficient around the periphery of non-circular channels, however.

In the study reported here, turbulent flow friction factor and forced convection heat transfer coefficient in a water-cooled, non-uniformly heated vertical channel with trapezoidal cross-section and rounded corners, were experimentally investigated. The objective was to examine the impact of heat flux non-uniformity on the aforementioned parameters.

## 2. Experiments

The experiments were performed in the Georgia Tech Large-Scale Thermal-Hydraulic Test Facility (GTLTTF). The test facility, described in [13], is a fully instrumented loop capable of providing steady-state flow rates of water at well-controlled and pre-adjusted conditions. For the present study, the test loop provided water flow rates at 0.063–0.505 l/s at pre-adjusted temperatures in the 30–70°C range, to the test section, with the test section back pressure varying in the 100–780 kPa range.

The test section was 139.4 cm in length, had a hydraulic diameter of 1.14 cm, and a 60.9 cm-long

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Nomenclature	
$C_p$	Specific heat (J/kg K)
$D_h$	Hydraulic diameter (m)
$f$	Friction factor
$G$	Mass flux (kg/m <sup>2</sup> s)
$Gr$	Grashof number = $\frac{(\Delta\rho/\rho)gD_h^3}{(\mu/\rho)^2}$
$h$	Heat transfer coefficient (W/m <sup>2</sup> K)
$k$	Thermal conductivity (W/m K)
$L$	Channel length (m)
$Nu$	Nusselt number = $hD_h/k$
$P$	Pressure (N/m <sup>2</sup> )
$\Delta P$	Pressure drop (N/m <sup>2</sup> )
$Pr$	Prandtl number = $\mu C_p/k$
$q''$	Heat flux (W/m <sup>2</sup> )
$Re$	Reynolds number = $GD_h/\mu$
$S$	Azimuthal coordinate (m)
$S'$	Channel perimeter (m)
$T$	Temperature (K)
<i>Greek symbols</i>	
$\epsilon$	Surface roughness (m)
$\mu$	Viscosity (kg/ms)
$\rho$	Density (kg/m <sup>3</sup> )
<i>Subscripts</i>	
f	Friction, bulk coolant
W	Wall
<i>Superscript</i>	
-	Average

segment of its top was heated, as described below. The length-to-hydraulic diameter ratio for the unheated segment was thus 69, which is believed to be sufficient to ensure fully developed hydrodynamics in the heated segment. The wall temperature measurements furthermore, were conducted at distances equal to 12.7 and 22 mm upstream from the test section exit, resulting in a length-to-hydraulic diameter ratio of more than 50 for the heated segment, which is also believed to be sufficient for thermally-developed conditions. The test section exit was connected to a 5 cm diameter pipe.

Fig. 1(a) displays the cross-section of the test section. The test section was produced by applying electrical discharge machining (EDM) to two stainless steel bars, each 679.5 mm long. (This length was the limit on the maximum length which can be machined by EDM, and necessitated using two pieces). Fourteen 3.18 mm-diameter semi-circular grooves, 60.96 cm long, were cut into the bars; the grooves in the bar forming the upper half of the test section housed 28 cartridge heaters, each 29.21 cm long (two heaters in each groove). The cross-section of the test section assembly is displayed in Fig. 1(b). Accounting for symmetry, the heaters can be divided into seven groups, each having four identical heaters. By adjusting the power to the individual heating elements, the desired wall heat flux distribution could be obtained.

Two groups of type-E thermocouples were used to measure the temperature of the test section wall. These thermocouples were housed in holes which were drilled to within 0.48–0.79 mm from the flow channel inner surface. Three thermocouples were distributed around the test section periphery 22 mm upstream from the exit of the heated segment; the other seven thermocouples were distributed as depicted in Fig. 1(a), at 12.7 mm from the exit.

The heated channel experiments were meant to represent axially uniform heating, with an azimuthal heat flux distribution similar to Fig. 2, which depicts a typical power distribution in the prototypical accelerator production of tritium (APT) channel [14]. In all experiments with non-uniform heating, the power distribution was similar to Fig. 2. The finite-element computer code ANSYS® [15] was used for the adjustment of the power distribution among the test section heaters, and for further analysis of the data. Since the axial power distribution was uniform, and bearing in mind that the heated segment of the test section is sufficiently long, two-dimensional calculations were performed with ANSYS. In the calculations for power adjustment, the power input to the heaters (which were simulated assuming uniform volumetric heating in them), the bulk coolant temperature, and the convection heat transfer coefficient (assumed to be azimuthally uniform, and calculated using the correlation of Dittus and Boelter [16]) were assumed known, and the distribution of wall heat flux was calculated by the code.

ANSYS was also used for analyzing the heat transfer data. These calculations involved an iterative solution, and were performed using a Digital Visual FORTRAN program that was written to use ANSYS as a subroutine. In each iteration, the measured wall temperatures, along with the test section outer surface boundary conditions (assumed to be in natural convection with surrounding air) and heater power input distribution were used to determine the surface heat flux and surface temperature distribution around the channel perimeter. These, in turn, together with local coolant bulk temperature, were used to determine the local heat transfer coefficients around the channel perimeter. The sensitivity of the results to the test section outer surface boundary conditions was examined by performing

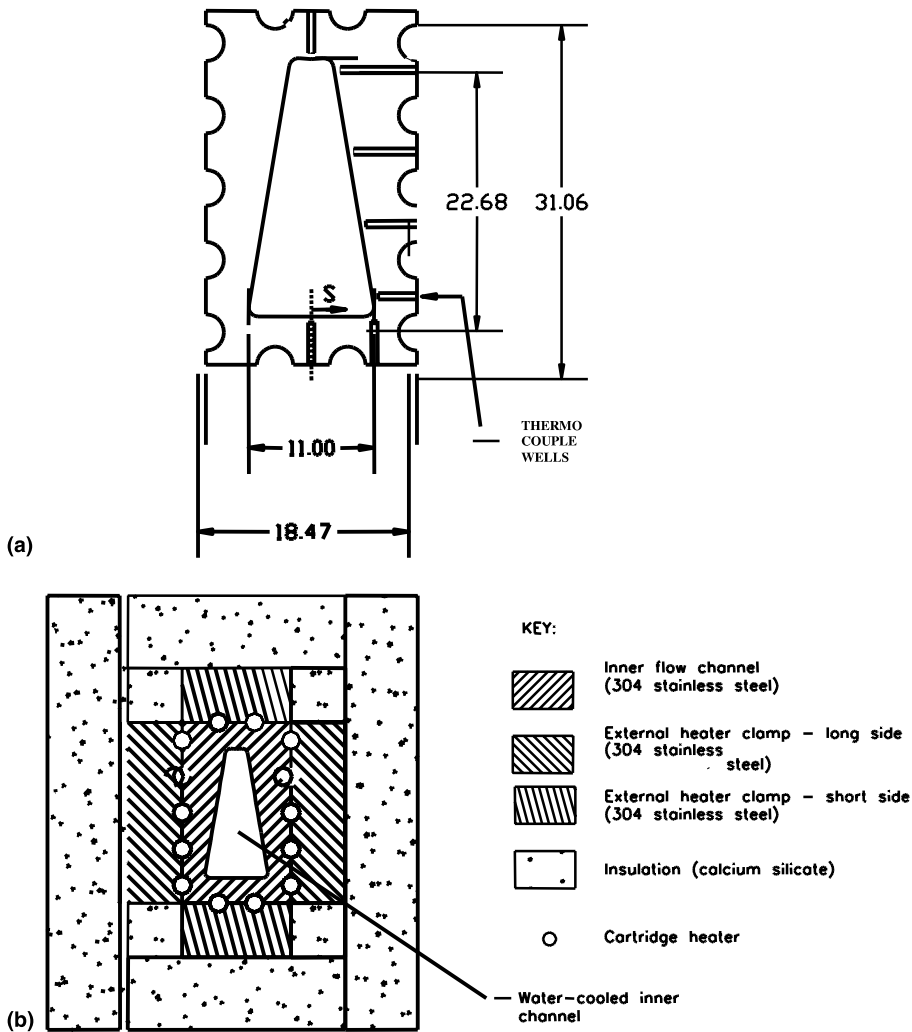


Fig. 1. The test section: (a) cross-section of the test section and locations of thermocouples at 12.7 mm upstream from the test section exit (not to scale); (b) cross-section of the test section and the heaters.

calculations using heat transfer coefficients found from appropriate natural convection correlations, and repeating the calculations using ten times higher heat transfer coefficients. The changes in calculated results were negligibly small. Further details can be found in [13].

In the experiments, the smallest value of the parameter  $Gr/Re^2$  (calculated based on the peak local heat flux, at lowest flow rate) was about  $10^{-3}$  when  $D_h$  was used as the length scale for both  $Gr$  and  $Re$ , and  $10^{-1}$  when the heated length was the length scale, thus ensuring that natural convection effect was always negligible.

Based on a detailed uncertainty analysis, the estimated average errors in the local heat transfer coefficients were within 1.5–6%.

### 3. Results and discussion

Fig. 3 displays the experimental values of the friction factor for heated and unheated test section conditions, based on the channel hydraulic diameter ( $D_h = 1.14$  cm), according to:

$$\Delta P_f = \bar{f} \frac{L}{D_h} \frac{G^2}{2\rho}, \quad (1)$$

where  $\bar{f}$  and  $\bar{\rho}$  represent the average friction factor and density, respectively. The predictions of the correlations of Blasius [17], for smooth channel, and Colebrook [18] for rough channel, are also depicted in the figure. The depicted rough-wall calculations are based on the assumption of a constant  $\epsilon/D_h = 5.6 \times 10^{-4}$ , which is

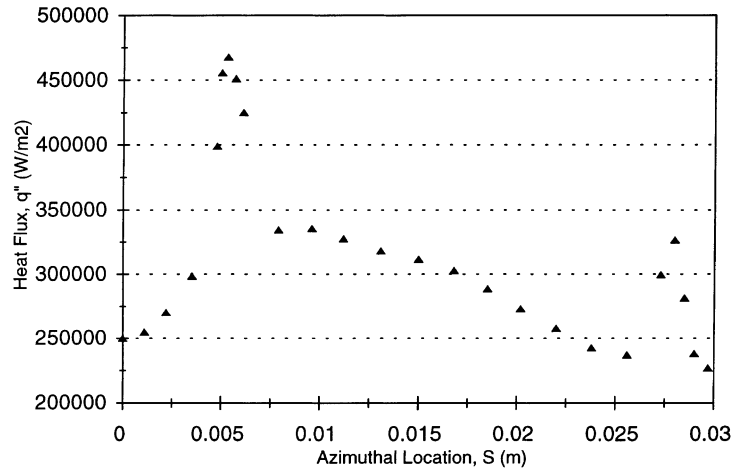


Fig. 2. Typical azimuthal heat flux distribution.

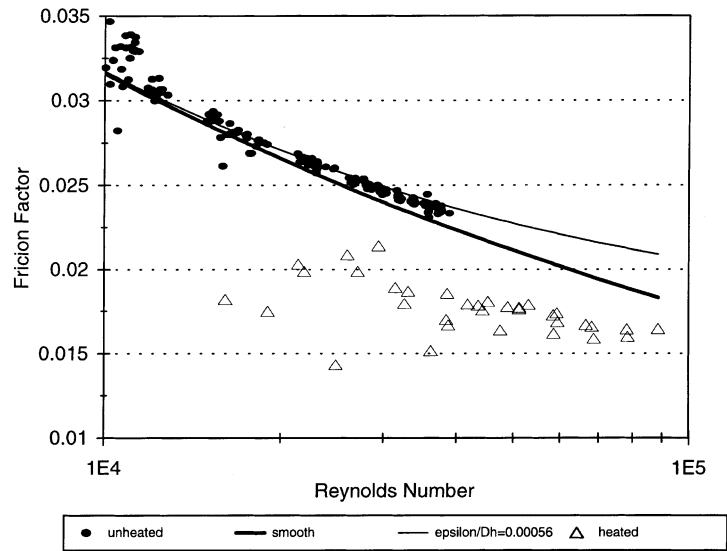


Fig. 3. The experimental friction factors compared with the correlation of Colebrook [17].

based on a reported value of  $\epsilon$  for the EDM process used to manufacture the test section. The Colebrook correlation evidently well predicts the data obtained with the test section unheated.

Friction factor data for the heated experiments, as noted in Fig. 3, are consistently lower than those predicted by the correlation of Colebrook [18] typically by about 30%, likely due to the occurrence of large temperature differences between the fluid and channel walls. In view of the sensitivity of water viscosity to temperature, and given the fact that Colebrook's correlation predicts the unheated test section data well, these results suggest that the use of the average bulk temperature to determine the fluid properties used for the calculation of

experimental friction factors (Eq. (1)), and for the calculations based on the aforementioned empirical correlations, may not be appropriate.

For each experiment, the local heat transfer coefficient values around the periphery were determined using the distribution of local surface temperature and heat flux, and the bulk fluid temperature, all at one axial location. (Note that the heat flux does not vary axially.) Typical results showing the normalized local Nusselt number variations around the perimeter of the test section are depicted in Fig. 4, where local  $Nu$  values are normalized with respect to the average  $Nu$  around the perimeter. The sensitivity of the measured  $Nu$  distributions to the variations in  $Re$  are also displayed in Fig. 4,

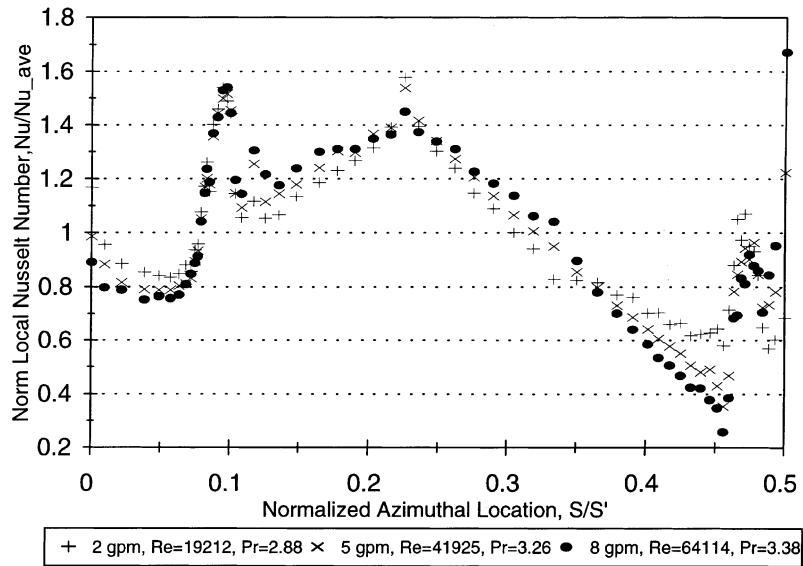


Fig. 4. Azimuthal distribution of Nusselt number ( $T_f \approx 50^\circ\text{C}$ ).

and indicate a relatively weak dependence of local  $Nu$  on  $Re$ . Similar sensitivity tests indicated that measured local  $Nu$  values were almost independent of  $Pr$ .

A total of 34 experiments were carried out, with azimuthally non-uniform heat flux distributions similar to Fig. 2. The local heat transfer coefficients discussed above were used to calculate the average heat transfer coefficients in two ways. In one method, the average heat transfer coefficient,  $\bar{h}$ , was found from:

$$\bar{h} = \frac{1}{S'} \int_0^{S'} h(S) \, dS. \quad (2)$$

In the second method, the average heat transfer coefficient,  $\bar{h}_T$ , was calculated from:

$$\bar{h}_T = \bar{q}'' (\bar{T}_W - T_f), \quad (3)$$

where  $T_f$  is the local coolant bulk temperature, and  $\bar{T}_W$  and  $\bar{q}''$  are the local perimeter-average wall temperature and heat flux, respectively, with  $\bar{T}_W$  found from:

$$\bar{T}_W = \frac{1}{S'} \int_0^{S'} T_W(S) \, dS. \quad (4)$$

The quantity  $\bar{q}''$  is defined similarly.

Figs. 5(a) and (b) compare the experimental average heat transfer coefficients  $\bar{h}$  and  $\bar{h}_T$ , respectively, with the predictions of three widely used correlations, all for the plane 12.7 mm from the test section exit (where seven thermocouples were installed), where the total and heated length-to-diameter ratios were 121 and 52, respectively. Since the power distribution in the axial direction was uniform, comparison of the data with correlations representing fully developed conditions can

be justified. The correlations utilized for comparison are due to Dittus and Boelter [16], Gnielinski [19], and Sieder and Tate [20], depicted in the forthcoming Eqs. (5)–(7), respectively:

$$Nu = 0.023 Re^{0.8} Pr^{0.4}, \quad (5)$$

$$Nu = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}, \quad (6)$$

$$Nu = 0.027 Re^{0.8} Pr^{0.33} (\mu/\mu_w)^{0.14}. \quad (7)$$

Gnielinski's correlation is to be used with the following friction factor, proposed by Filonenko [21]:

$$f = (1.82 \log_{10} Re - 1.64)^{-2}. \quad (8)$$

Also,  $\mu_w$  in Eq. (7) is obtained using  $\bar{T}_W$  provided by Eq. (4).

The scatter in the normalized heat transfer coefficients in Figs. 5(a) and (b) is caused by the changes in  $Pr$  for data with comparable  $Re$  values. The trends in Figs. 5(a) and (b) are similar: all three correlations tend to under predict the data over the entire range of parameters tested. In Fig. 5(a), the correlation of Dittus and Boelter [16] under predicts the data by about 28%, although at high  $Re$  ( $Re \sim 8 \times 10^4$ ) the under prediction of the data is by as much as 66%. The correlation of Gnielinski [19] under predicts the data by 17% on the average, and by 47% maximum, and the corresponding values for the correlation of Sieder and Tate [20] are 11% and 51%, respectively. The extent by which these correlations under predict  $\bar{h}_T$ , as depicted in Fig. 5(b), is only slightly smaller than those for  $\bar{h}$ .

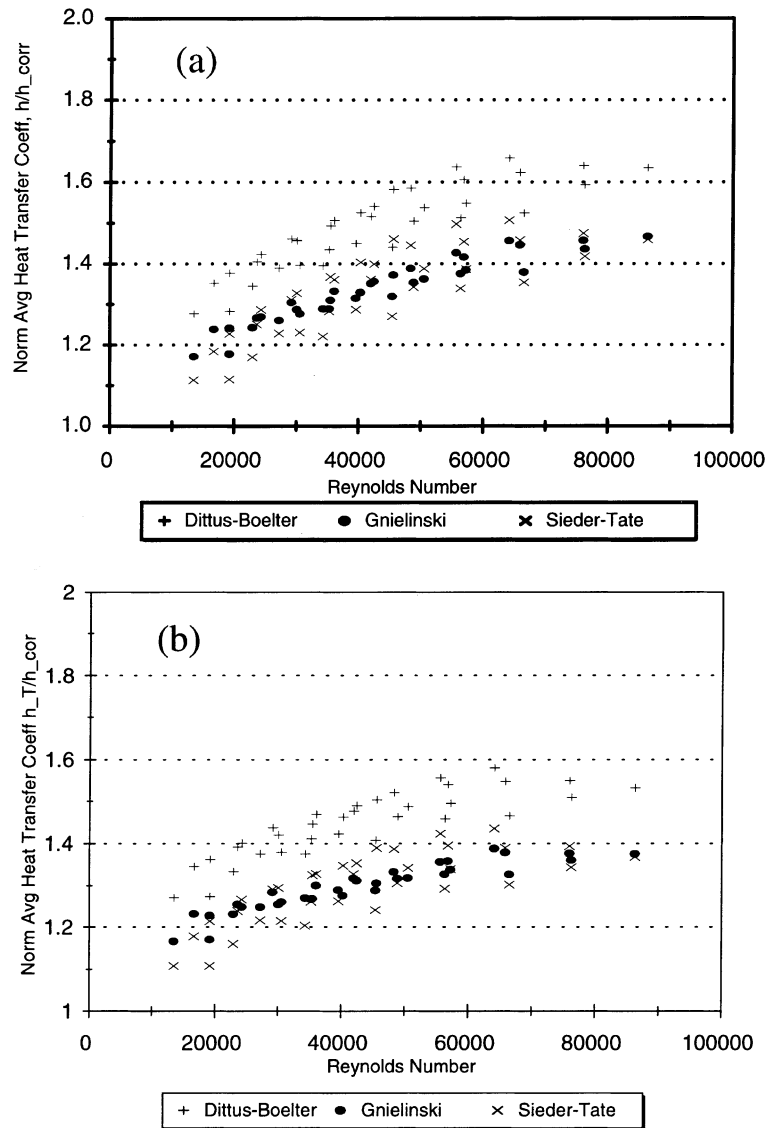


Fig. 5. Experimental heat transfer coefficients compared with various correlations: (a)  $\bar{h}$ ; (b)  $\bar{h}_1$ .

#### 4. Conclusions

Turbulent flow wall friction and convection heat transfer in a trapezoidal channel were experimentally studied, using a test section with a hydraulic diameter of 1.14 cm and total and heated lengths of 139.4 and 60.9 cm, respectively.

The correlation of Colebrook [18] well predicted the friction factors for unheated experiments and systematically over predicted the measured friction factors when the test section was heated.

Perimeter-average convection heat transfer coefficients for the exit of the channel (where fully developed

conditions are justified) were obtained in experiments with laterally non-uniform wall heat fluxes. The correlations of Dittus and Boelter [16], Gnielinski [19] and Sieder and Tate [20] all systematically under predicted the data, typically by 11–28%.

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